Superbranes, $D = 11$ CJS supergravity and enlarged superspace coordinates/fields correspondence[*]

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Abstract

We discuss the rôle of enlarged superspaces in two seemingly different contexts, the structure of the p-brane actions and that of the Cremmer-Julia-Scherk eleven-dimensional supergravity. Both provide examples of a common principle: the existence of an *enlarged superspaces coordinates/fields correspondence* by which all the (worldvolume or spacetime) fields of the theory are associated to coordinates of enlarged superspaces. In the context of p-branes, enlarged superspaces may be used to construct manifestly supersymmetry-invariant Wess-Zumino terms and as a way of expressing the Born-Infeld worldvolume fields of D-branes and the worldvolume M5-brane two-form in terms of fields associated to the coordinates of these enlarged superspaces. This is tantamount to saying that the Born-Infeld fields have a superspace origin, as do the other worldvolume fields, and that they have a composite structure. In $D=11$ supergravity theory enlarged superspaces arise when its underlying gauge structure is investigated and, as a result, the composite nature of the A_3 field is revealed: there is a full one-parametric family of enlarged superspace groups that solve the problem of expressing A_3 in terms of spacetime fields associated to their coordinates. The corresponding enlarged supersymmetry algebras turn out to be deformations of an *expansion* of the $osp(1|32)$ algebra. The unifying mathematical structure underlying all these facts is the cohomology of the supersymmetry algebras involved.

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1 Introduction

M-theory (see [1, 2] and [3] for a chronological history) is not based at present on a definite
Lagrangian or on an S-matrix description; rather, its conjectured existence relies on the
properties of its six perturbative and low energy limits (string models and supergravities)
and by dualities [4] among them. Such dualities, including those relating apparently differ-
ent models, are believed to be symmetries of M-theory. The full set of M-theory symmetries
-as the full M-theory itself- is not known[1], but it should include these dualities as well as
the symmetries of the different superstring and supergravity limits. For this reason, the
study of the symmetries of p-branes as well as the underlying gauge symmetry of $D = 11$
supergravity may help to understand the symmetry structure of M-theory itself.

Superalgebras going beyond the standard supersymmetry algebra were considered very
early (see [20, 21] and references in [22]) and, later, in the context of brane theory. Some
of these *enlarged supersymmetry algebras* generalize Green's algebra [23]. They were in-
troduced in [24] to make *Lie* algebras out of the free differential algebras that had been
introduced in [25] to recover cohomologically the classification [26] of the scalar *p*-branes.
The authors of [24] also showed that these algebras could be used to obtain Wess-Zumino
(WZ) terms for the rigid *p*-brane actions strictly invariant under supersymmetry. The
relation between semi- or quasi-invariance (*i.e.*, invariance but for a total derivative) of
lagrangians, cohomology and group extension theory, is a problem which has a fifty years
long history, but we will not discuss it here (see [27] and references therein). In the case
of *p*-branes, the additional variables of the new supersymetry groups [22] (rigid enlarged
superspaces) appear in these manifestly invariant WZ terms in a trivial way, but this is not
the case for all types of branes. It was shown in [22] that the enlarged superspaces, could
also be used to obtain Born-Infeld (BI) fields from one-forms defined on them (for BI fields
in the IIB case see [28]). In the case of the D-branes BI fields or with the worldvolume
two-form of the M5 brane these fields allow for the existence of actions where all world-
volume fields entering in the theory are associated to the enlarged superspace coordinates
(there are no fields 'external' to the superspace coordinates, *i.e.* *directly* defined on the
worldvolume). This points out to the existence of an *extended superspace coordinates/fields
correspondence* for branes [22], where in this case 'fields' refers to worldvolume fields. We
shall devote the first two sections to review these ideas.

It turns out that, in an analogous fashion, a similar correspondence may also be estab-
lished for $D = 11$ Cremmer-Julia-Scherk (CJS) [29] *supergravity*, in which case the fields

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[1]Several groups may play a rôle, as the rank 11 Kac-Moody E_{11} group [5] as a basis for a non-linear
realization approach to $D = 11$ supergravity as well as other Kac-Moody symmetries, or the $OSp(1|64)$
group [6, 7] and its subgroup $GL(32)$ [8, 9]. This last one is the automorphism group of the M-algebra
$\{Q_\alpha, Q_\beta\} = P_{\alpha\beta}$; it is also a manifest symmetry of the actions [10, 11] for BPS preons [12] (see [13] for
a review), the hypothetical constituents of M-theory. Clearly, in $D = 11$ supergravity one may see only
a fraction of the M-theory symmetries. As it was noticed recently [14, 15] (see also [11]), a suggestive
analysis of supersymmetric $D = 11$ supergravity solutions can be carried out in terms of generalized
connections with holonomy group $SL(32)$ (see [16] for an early reference on generalized holonomy). The
case for a $OSp(1|32) \otimes OSp(1|32)$ gauge symmetry in a Chern-Simons context was presented and discussed
in [17, 18, 19].

are spacetime fields. This correspondence between enlarged superspace coordinates and spacetime fields for CJS supergravity is related to the problem of its 'hidden' or underlying gauge symmetry. This problem was raised already in the original CJS paper [29]. It was considered by D'Auria and Fré [21] as a search for a composite structure of the *three–form* A_3 field that enters in the $D = 11$ supergravity multiplet (see also [30, 31] for other discussions of the geometry of $D = 11$ supergravity and [32] for an overview on local super-symmetry[2]). While the graviton, the gravitino and the spin connection are one-form fields, $e^a = dx^\mu e^a_\mu(x)$, $\psi^\alpha = dx^\mu \psi^\alpha_\mu(x)$ and $\omega^{ab} = dx^\mu \omega^{ab}_\mu(x)$, and can be considered as gauge fields for the superPoincaré group [36], the fully antisymmetric $A_{\mu_1\mu_2\mu_3}(x)$ (transverse) abelian gauge field is not associated with a symmetry generator and it rather corresponds to a *three*-form A_3 on spacetime. This prevents the association of the fields of the standard $D = 11$ supergravity multiplet with the gauge fields of a Lie superalgebra, since these are associated to *one*-forms.

Two enlarged supersymmetry algebras with 528 bosonic and 64 fermionic generators

$$P_a \, , \, Q_\alpha \, ; \, Z_{a_1a_2} \, , \, Z_{a_1...a_5} \, ; \, Q'_\alpha \, , \tag{1}$$

including the 528+32 M-algebra [2] ones plus a central fermionic generator Q'_α, were found in [21] to allow for a decomposition of A_3. The corresponding one-form fields

$$e^a \, , \, \psi^\alpha \, ; \, B^{a_1a_2} \, , \, B^{a_1...a_5} \, , \, \eta^\alpha \, , \tag{2}$$

were then considered as gauge fields for these larger supergroups. In this scheme, all the CJS supergravity fields can then be treated as gauge fields, with A_3 expressed in terms of them.

As we shall see, the problem studied in [21] is mathematically analogous to that of obtaining strictly invariant WZ terms for p-branes from the originally quasi-invariant (invariant up to a total derivative) ones. Both reduce to finding a Lie algebra allowing us to write a closed invariant form (on the original supergroup manifold) as the differential of an invariant one (now on the manifold of the associated enlarged supergroup). Expressed in another way, these problems correspond to finding a trivialization of certain non-trivial Chevalley-Eilenberg (CE) [37] cocycles for the cohomology of the standard supersymme-try algebra of the theory by means of enlarging it. It turns out that the underlying gauge supergroup structure of $D = 11$ CJS supergravity can be described by any representative of a *one–parametric* family of supergroups denoted $\tilde{\Sigma}(s)$ ($s \neq 0$), or by their associated superalgebras $\tilde{\mathfrak{E}}(s)$, the two D'Auria-Fré ones being two particular elements of that family (specifically, $\tilde{\mathfrak{E}}(3/2)$ and $\tilde{\mathfrak{E}}(-1)$). There have been attempts to relate these solutions to some known algebra (see [38]). We will see that the algebras $\tilde{\mathfrak{E}}(s)$ are nontrivial ($s \neq 0$) deformations of the special element $\tilde{\mathfrak{E}}(0)$. The $\tilde{\mathfrak{E}}(s), s \neq 0$, automorphism group is $SO(1, 10)$. Thus, the relevant supergroup that replaces the standard superPoincaré group $\Sigma \rtimes SO(1, 10)$ becomes the semidirect product $\tilde{\Sigma}(s) \rtimes SO(1, 10)$, a deformation of $\tilde{\Sigma}(0) \rtimes SO(1, 10)$. As for the superalgebra $\tilde{\mathfrak{E}}(0) \ni so(1, 10)$ itself, it is related to $osp(1|32)$

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[2]For recent discussions in a different perspective see [33, 34, 35].

through an *expansion*[3]. Specifically,

$$\tilde{\Sigma}(0) \otimes SO(1,10) \approx OSp(1|32)(2,3,2) \,, \tag{3}$$

where the numbers in $OSp(1|32)(2,3,2)$ characterize the expansion (see later). For $s = 0$ the $SO(1,10)$ automorphism group is enhanced to $Sp(32)$, and one finds that $\tilde{\Sigma}(0) \otimes Sp(32) \approx OSp(1|32)(2,3)$.

The supergroup manifolds $\tilde{\Sigma}(s)$ determine *rigid, enlarged* superspaces. The fact that all the spacetime fields in (2) may be associated to the various coordinates of the $\tilde{\Sigma}(s)$ supergroups again suggests that there is an *extended superspace coordinates/fields correspondence* principle[4].

2 Wess-Zumino terms for super-p-branes and enlarged supersymmetry algebras

In this section, and in the next one, we describe the rôle of enlarged superspaces in brane theory. We will start from the scalar p-branes [26] case. The action for a p brane in rigid superspace is given by the sum of two terms,

$$I = I_0 + I_{WZ} \,, \tag{4}$$

where I_0 is the kinetic part and $I_{WZ} = \int_{\mathcal{W}} \phi^*(b)$ is the WZ term, which is given by the integral over the worldvolume $\mathcal{W}$, parametrized by $\xi^i = (\tau, \sigma^1, \dots, \sigma^p)$ $[i = 0, \dots, p]$ of the pull-back $\phi^*(b)$ to $\mathcal{W}$ of a $(p+1)$-form b defined on the rigid superspace $\Sigma^{(D|n)}$ (Σ for short) of the theory (the manifold of the corresponding supersymmetry group). This form b is the potential form of a $(p+2)$-form h which happens to be exact,

$$h = db \,, \tag{5}$$

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[3]The *expansion method* allows us to obtain new algebras from a given one, in general of higher dimension than the original one. Under a different name, expansions were considered in [39], and the method was studied in general in [40].

[4]The idea of a 'fundamental symmetry between coordinates and fields' is explicitly stated in Berezin [41] and is implicit in earlier work of D. V. Volkov [42]; the field space democracy is also discussed in [43]. However, we are not referring here to a democracy between the fields and *their* arguments (as one might introduce by *e.g.*, considering on an equal footing the coordinates of the *total* space of a fibre bundle the cross sections of which may be used to define fields on the *base* manifold), but rather to a correspondence between the (enlarged) superspace coordinates and the fields they originate, be them worldvolume or spacetime ones. This is why is more precise to speak of a *correspondence* between (enlarged superspace) coordinates and fields rather than of 'democracy' -the term used in [22]- since its original use referred to a democracy between the fields and *its* arguments. We have conjectured [22] the existence of a correspondence between the *coordinates* of a suitable superspace and the *fields* in theory constructed on it. These appear as the pullbacks of forms, originally defined on the target enlarged superspace, to the worldvolume or spacetime manifolds. Also, the basis for such a correspondence is group theoretical: the enlarged rigid superspaces are all supergroup manifolds.

and that is invariant under the transformations of the superPoincaré group $\Sigma \otimes SO(1, D-1)$. The study of the different possible $(p + 2)$-forms h in superspaces corresponding to the minimal supersymmetries in D dimensions determines the (D, p) values for which the WZ term exists. With the appropriate relative factor, I_{WZ} in (4) leads to super-p-brane κ-invariant actions. These (D, p) values determine the 'old branescan' [26].

It turns out [25] that the h's are non-trivial Chevalley-Eilenberg (CE) $(p + 2)$-cocycles for the cohomology of the standard $\mathfrak{E}^{(D|n)}$ supersymmetry algebra. This means that h is a closed (obviously, $dh = 0$) and supersymmetry invariant $(p + 2)$-form constructed from the Maurer-Cartan (MC) one-forms on the graded translations (supersymmetry) group Σ (namely Π^μ and Π^α, where $\mu = 0, \ldots D - 1$, and the range $1, \ldots n$ of α depends on the minimal spinor considered). This CE cocycle condition depends on the known gamma matrix identities that are true only for the (D, p) values of the 'old branescan'. Furthermore, the non-triviality of these cocycles means that the potential $(p + 1)$-form b in $h = db$ is not supersymmetry invariant $i.e.$, that cannot be constructed from the invariant MC forms Π^μ and Π^α on Σ. An important consequence of this fact is that the WZ lagrangian is not manifestly invariant under supersymmetry, but only quasi-invariant (hence its 'WZ' name) and that, as a result, the algebra of charge densities produces topological extensions of the original supersymmetry algebra [44].

Certain enlarged rigid superspaces associated to enlarged supersymmetry groups $\tilde{\Sigma}$, with additional bosonic and fermionic variables, can be used to obtain p-brane actions that are equivalent to the standard ones but with WZ terms that are strictly invariant under supersymmetry. On the manifolds of these groups, the same $(p + 2)$-forms h of the old branescan are still CE cocycles, but they are now trivial ones: $h = d\tilde{b}$, and the new potential $(p + 1)$-forms $\tilde{b}$ are $\tilde{\Sigma}$-invariant. The process of obtaining these enlarged algebras or 'brane algebras' may be thus called of 'trivialization of the CE cocycles' h on $\mathfrak{E}^{(D|n)}$.

Let us look in more detail how to achieve this trivialization (the following is not, as we shall see, the only possibility). One starts with the MC equations of the standard supersymmetry algebra in D dimensions $\mathfrak{E}^{(D|n)}$ (we consider, for simplicity, the cases that allow for real spinors; wedge products are understood in this and in the next section)

$$d\Pi^\alpha = 0 \ , \quad d\Pi^\mu = a_s \Pi^\alpha (C\Gamma^\mu)_{\alpha\beta} \Pi^\beta \ , \tag{6}$$

where [5] the constant $a_s = 1/2$ for $\tilde{\mathfrak{E}}$. The $(p + 2)$-form h for a p-brane may be shown to be, up to a proportionality constant which is not important in the discussion below,

$$h = \Pi^\alpha (C\Gamma_{\mu_1 \ldots \mu_p})_{\alpha\beta} \Pi^\beta \Pi^{\mu_1} \ldots \Pi^{\mu_p} \tag{7}$$

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[5] *A comment on conventions.* The constant a_s is real since we assume real gamma matrices (which in $D = 11$, for instance, requires mostly plus metric) and the convention used here for the complex conjugation is $(\theta_1 \theta_2)^* = (\theta_1^* \theta_2^*)$, θ_1 and θ_2 being Grassmann odd. If we used the conjugation that reverses the order, as it will be the case in Secs. 4-7, the a_s in eq. (6) would be purely imaginary. Other differences in conventions between Secs. 2,3 and Secs. 4-7 are that in Secs.2,3 we write explicitly the charge conjugation matrix C; also, in Secs. 2,3 the $\wedge$ product for forms is implicit. We have kept these two sets of conventions in order to make direct contact with [22] and with [45].

which is closed for the dimensions D for which the identity

$$(C\Gamma^{\mu_1...\mu_p})_{(\alpha\beta}(C\Gamma_{\mu_1})_{\gamma\delta)} = 0 \tag{8}$$

is satisfied. The bilinear in (7) suggests that in order to find an invariant potential form for h one should first extend (6) adding the form $\Pi^{\mu_1...\mu_p}$ and the MC equation

$$d\Pi_{\mu_1...\mu_p} = a_0\Pi^\alpha(C\Gamma_{\mu_1...\mu_p})_{\alpha\beta}\Pi^\beta \ , \tag{9}$$

so that h is the first term in the differential of

$$\Pi_{\mu_1...\mu_p}\Pi^{\mu_1}\ldots\Pi^{\mu_p} \quad . \tag{10}$$

Before going to the next step, let us note that this is a sensible thing to do for two reasons. The first is that eq. (9) and the second of eqs. (6) can be put on the same footing since they are central (if one ignores the Lorentz part) extensions of the abelian odd translation algebra defined by the simple MC equations $d\Pi^\alpha = 0$. So if the graded supertranslations (supersymmetry) algebra is itself a central extension, it seems mathematically natural to consider other possible extensions as well. In fact, one should consider the most general extension where the 'central' generators appear for each symmetric $(C\Gamma^{\mu_1...\mu_q})$ matrix (we shall keep, here, however, only the generators corresponding to Π^μ and one of the $\Pi^{\mu_1...\mu_p}$ for simplicity; they will be sufficient to discuss the 'scalar' brane actions). The second reason is that including these new bosonic generators is necessary to understand, from the algebraic point of view, the existence of BPS states that break some supersymmetries but not all, as known from supergravity theories.

Equation (10) does not solve yet the problem of finding an invariant $\tilde{b}$ such that $d\tilde{b} = h$ because the exterior differential also acts on the p factors Π^μ . This means that new generators and MC equations have to be added to (6) and (10). It may be shown (see [24] and [22]) that such a $\tilde{b}$ can be found if the algebra is extended in several steps, each step involving a central (if we ignore the Lorentz part) extension of the algebra resulting from the previous one. The first invariant form by which one extends is fermionic, and has the structure $\Pi_{\mu_1...\mu_{p-1}\alpha_1}$ (so that, for $p = 1$, one obtains the Green algebra [23]). The second is an extension of the algebra whose MC equations are generated by Π^α, Π^μ, $\Pi_{\mu_1...\mu_p}$ and $\Pi_{\mu_1...\mu_{p-1}\alpha_1}$, and the new invariant forms have the structure $\Pi_{\mu_1...\mu_{p-2}\alpha_1\alpha_2}$. This process of extensions ends when the last invariant form $\Pi_{\alpha_1...\alpha_p}$ is added. At each step in the above procedure the extension made is central, but it makes non-central the former central generator of the previous step. Thus, the resulting algebra is not a central extension of the supersymmetry one but for $p = 1$ where the only step produces the Green algebra [23]. The existence of these enlarged supersymmetry or 'brane algebras' depends on the values of D and p for which the identity (8) holds; this is not surprising since these algebras allow for the existence of $\tilde{b}$ such that $d\tilde{b} = h$ and $dh = 0$ is true only when (8) is satisfied.

We shall not give here the explicit expressions for the resulting 'brane algebras' in general; these can be found in [24] and in [22]; the associated enlarged superspace groups law is also given in [22] for the most interesting ones. We shall only write explicitly two of

these enlarged supersymmetry algebras, because they will be relevant in the next sections. Let us begin by the superalgebra for $p = 2$, $D = 11$ that trivializes the CE cocycle that defines the WZ term of the $D = 11$ membrane [46]. It is given by the MC equations

$$
\begin{aligned}
d\Pi^\alpha &= 0 , \quad d\Pi^\mu = \frac{1}{2}(C\Gamma^\mu)_{\alpha\beta}\Pi^\alpha\Pi^\beta , \\
d\Pi^{\mu\nu} &= \frac{1}{2}(C\Gamma^{\mu\nu})_{\alpha\beta}\Pi^\alpha\Pi^\beta , \\
d\Pi_{\mu\alpha} &= (C\Gamma_{\nu\mu})_{\alpha\beta}\Pi^\nu\Pi^\beta + (C\Gamma^\nu)_{\alpha\beta}\Pi_{\nu\mu}\Pi^\beta , \\
d\Pi_{\alpha\beta} &= -\frac{1}{2}(C\Gamma_{\mu\nu})_{\alpha\beta}\Pi^\mu\Pi^\nu - \frac{1}{2}(C\Gamma^\mu)_{\alpha\beta}\Pi_{\mu\nu}\Pi^\nu \\
&\quad + \frac{1}{4}(C\Gamma^\mu)_{\alpha\beta}\Pi_{\mu\delta}\Pi^\delta + (C\Gamma^\mu)_{\delta\alpha}\Pi_{\mu\beta}\Pi^\delta + (C\Gamma^\mu)_{\delta\beta}\Pi_{\mu\alpha}\Pi^\delta .
\end{aligned}
\tag{11}
$$

They can be obtained from the general expressions of [22], particularized to the case $p = 2$, by suitably fixing the undetermined constants. The above equations allow for the existence of an invariant $\tilde{b}$ such that $d\tilde{b} = h = \Pi^\alpha(C\Gamma^{\mu\nu})_{\alpha\beta}\Pi^\beta\Pi_\mu\Pi_\nu$. The expression for $\tilde{b}$ is [24, 22]

$$
\tilde{b} = \frac{2}{3}\Pi_{\mu\nu}\Pi^\mu\Pi^\nu - \frac{3}{5}\Pi_{\mu\alpha}\Pi^\mu\Pi^\alpha - \frac{2}{15}\Pi_{\alpha\beta}\Pi^\alpha\Pi^\beta .
\tag{12}
$$

The second algebra that we shall need is the one that trivializes the CE cocycle associated to the WZ term of the D=10, IIA superstring. It can be extracted from the dimensional reduction to $D = 10$ of the algebra (11) (see [22], eqs. (88) and (90)), and is given by

$$
\begin{aligned}
d\Pi^\alpha &= 0 , \quad d\Pi^\mu = \frac{1}{2}(C\Gamma^\mu)_{\alpha\beta}\Pi^\alpha\Pi^\beta \\
d\Pi^{(z)}_\mu &= \frac{1}{2}(C\Gamma_\mu\Gamma_{11})_{\alpha\beta}\Pi^\alpha\Pi^\beta \\
d\Pi^{(z)}_\alpha &= (C\Gamma_\nu\Gamma_{11})_{\alpha\beta}\Pi^\nu\Pi^\beta + (C\Gamma^\nu)_{\alpha\beta}\Pi^{(z)}_\nu\Pi^\beta ,
\end{aligned}
\tag{13}
$$

where the superscript (z) refers to the fact that the forms $\Pi^{(z)}_\mu$ and $\Pi^{(z)}_\alpha$ come from the dimensional reduction of $\Pi_{\mu\nu}$ and $\Pi_{\mu\alpha}$ respectively when μ corresponds to the z coordinate in the splitting $x^0, \ldots, x^9, z$. This algebra is consistent due to the $D = 10$ gamma matrices identity

$$
(C\Gamma^\mu\Gamma_{11})_{(\alpha\beta}(C\Gamma_\mu)_{\gamma\delta)} = 0 .
\tag{14}
$$

The corresponding $\tilde{b}$ is given by

$$
\begin{aligned}
\tilde{b} &= \frac{1}{2}\Pi^\alpha\Pi^{(z)}_\alpha - \Pi^\mu\Pi^{(z)}_\mu , \\
d\tilde{b} &= h = (C\Gamma_\mu\Gamma_{11})_{\alpha\beta}\Pi^\mu\Pi^\alpha\Pi^\beta .
\end{aligned}
\tag{15}
$$

We note finally that the coordinates of $\tilde{\Sigma}/\Sigma$ [$(\varphi^{\mu\nu},\varphi_{\mu\alpha},\varphi_{\alpha\beta})$ for eq. (11) and $(\varphi^\mu, \varphi_\alpha)$ for eq. (13)] that, beyond the ordinary superspace $\Sigma^{(D|n)}$ ones (x^μ, θ^α), complete the

parametrization the enlarged superspaces $\tilde{\Sigma}$, lead to non-dynamical fields in the action. The WZ term is written in invariant form as $\phi^*(\tilde{b})$, where ϕ^* is the pullback that takes the form $\tilde{b}$ on the $\tilde{\Sigma}$ manifold to $\mathcal{W}$. Indeed, since $d\tilde{b} = h = db$, it follows that the new fields enter in the WZ part of the action $\int_{\mathcal{W}} \phi^*(\tilde{b})$ as total derivative[6]. However, they appear non trivially in the context of D-branes and in the M5-brane, as we discuss in the next section, where they also appear in the D-brane action kinetic part.

3 The enlarged superspace coordinates/fields correspondence for superbranes

The action for D-branes [48, 49, 50] (we shall restrict ourselves to the type IIA D-branes as in [22], see [28] for the IIB case) in a rigid background for which all forms in the R-R sector and the dilaton vanish is given, as in the case of p-branes (4), by the sum of a kinetic term I_0 and a WZ term I_{WZ}. The first one is

$$I_0 = \int d\xi^{p+1} \sqrt{-\det(g_{ij} + \mathcal{F}_{ij})} \quad . \tag{16}$$

In (16), g_{ij} is the induced metric on the worldvolume, $g_{ij}(\xi) = \Pi_i^\mu(\xi)\Pi_{\mu j}(\xi)$ ($\Pi^\mu = \Pi_i^\mu d\xi^i$), and $\mathcal{F}_{ij}$ are the worldvolume components of the form $\mathcal{F}(\xi) = dA_1(\xi) - \phi^*(B_2)$, where $\phi^*(B_2)$ is the pull-back to $\mathcal{W}$ of a two-form (B_2) defined on the $D = 10$, IIA superspace such that

$$dB_2 = -(C\Gamma_\mu\Gamma_{11})_{\alpha\beta}\Pi^\alpha\Pi^\beta\Pi^\mu , \tag{17}$$

and $A_1(\xi)$ is a one-form *directly* defined on the worldvolume, the BI field, that transforms in such a way that $\mathcal{F}$ is invariant under supersymmetry. The WZ term is quasi-invariant and is given by the integral of a $(p+1)$-form that depends polynomially on $\mathcal{F}$, the coefficients being forms on the $D = 10$ IIA superspace. The explicit expressions for the different D-brane WZ terms (actually for even $p = 2, 4, 6, 8$) are not relevant for our purposes, but we note here that the search for the possible non-trivial CE cocycles that determine h also identifies the possible D-branes [22] recovering Polchinski's classification [51] (for recent work on D-branes see [52]). Similarly, the $D = 11$ M5-brane action [53] [7] contains a *two*-form $A_2(\xi)$ directly defined on the worlvolume that enters the action through the field strength $H_3(\xi) = dA_2(\xi) - \phi^*(\mathcal{A}_3)$, where $\phi^*(\mathcal{A}_3)$ is the pull back to the worldvolume of a $D = 11$ superspace three-form $\mathcal{A}_3$ such that

$$d\mathcal{A}_3 = -(C\Gamma_{\mu\nu})_{\alpha\beta}\Pi^\mu\Pi^\nu\Pi^\alpha\Pi^\beta ; \tag{18}$$

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[6]We shall not discuss the behaviour of the additional variables under κ-symmetry, for which we refer to [47].

[7]The covariant equations for the D-branes and the M5-brane were found in [54] in the framework of the superembedding approach developed for the supermembrane and superstrings in [55].

$A_2(\xi)$ transforms under supersymmetry in such a way that H is invariant [8]. Again we note in passing that a CE cohomological search for the possible $D = 11$ WZ terms in this case leads to the M5 brane as the only solution [22].

In contrast with p-branes, both the D-brane and M5-brane actions cannot be written in terms of forms associated to the coordinates of ordinary superspaces, due to the presence of the one- and two-forms $A(\xi)$, which are defined directly on the worldvolume. The arguments of the previous section, however, lead to the possibility of writing them solely in terms of forms defined on suitably *enlarged* superspaces as we describe now.

Let us first consider the IIA D-branes case (this includes the D2, D4, D6 and D8 cases). The two-form $\mathcal{F}$ is supersymmetry invariant and has the property that $\mathcal{F} = \phi^*((C\Gamma_\mu\Gamma_{11})_{\alpha\beta}\Pi^\alpha\Pi^\beta\Pi^\mu)$. But these conditions are also satisfied by $\tilde{b}$ in eq. (15). Moreover, as discussed in the previous section, the new superspace variables appear in $\tilde{b}$ inside a total differential. So one may consider the IIA enlarged superspace defined by the MC equations (13) and identify $\mathcal{F}$ with $\phi^*(\tilde{b})$. Since $\tilde{b}$ has a part that is a total differential which contains the new superspace coordinates, one concludes that dA may be identified with this part. The result [22] is

$$A_1(\xi) = \phi^* \left(\varphi_\mu dx^\mu + \frac{1}{2}\varphi_\alpha d\theta^\alpha \right) . \tag{19}$$

In the M5-brane case, the relevant enlarged superspace is the one corresponding to the MC equations (11), and the expression for the two-form $A_2(\xi)$ that arises by identifying H with $\phi^*(\tilde{b})$ from eq. (12) turns out to be [22]

$$
\begin{aligned}
A_2(\xi) \ = \ \phi^* \Big(& \frac{2}{3}\varphi_{\mu\nu}dx^\mu dx^\nu - \frac{3}{5}\varphi_{\mu\alpha}dx^\mu d\theta^\alpha - \frac{2}{15}\varphi_{\alpha\beta}d\theta^\alpha d\theta^\beta + \frac{1}{30}\varphi_{\mu\nu}x^\mu(C\Gamma^\nu)_{\alpha\beta}d\theta^\alpha d\theta^\beta \\
& + \frac{11}{30}\varphi_{\mu\nu}dx^\mu(C\Gamma^\nu)_{\alpha\beta}\theta^\alpha d\theta^\beta - \frac{13}{180}\varphi_{\mu\nu}(C\Gamma^\mu)_{\alpha\beta}(C\Gamma^\nu)_{\delta\epsilon}\theta^\alpha d\theta^\beta\theta^\delta d\theta^\epsilon \\
& + \frac{1}{10}\varphi_{\mu\alpha}(C\Gamma^\mu)_{\delta\epsilon}\theta^\delta d\theta^\epsilon d\theta^\alpha + \frac{1}{20}\varphi_{\mu\alpha}(C\Gamma^\mu)_{\delta\epsilon}d\theta^\delta d\theta^\epsilon\theta^\alpha \Big) .
\end{aligned}
\tag{20}
$$

So far we have argued that $\phi^*(\tilde{b})$ has the same supersymmetry properties as $\mathcal{F}(\xi)$ (for the IIA D-branes) or $H(\xi)$ (for the M5 brane). The next question to ask is whether it is legitimate to substitute the latter for the former in the actions for the D-branes and the M5-brane. Let us begin by the D-brane case. It suffices to show that the Euler-Lagrange (E-L) equations for the actions that are obtained by replacing $A_1(\xi)$ by the r.h.s. of (19), which we shall denote more explicitly by $A_1(x(\xi), \theta(\xi), \varphi(\xi)) \equiv A_1(x, \theta, \varphi)$, have the same dynamical content as those for the original one, where $I = I[x, \theta, A_1]$. Indeed, $I[x, \theta, A_1(x, \theta, \varphi)]$ has the same variation as $I[x, \theta, A_1]$, except for the fact that one has to vary also the fields inside $A_1(x, \theta, \varphi)$. From the variation of $I[x, \theta, A_1(x, \theta, \varphi)]$ with respect to the new variables, $\varphi = (\varphi_\mu, \varphi_\alpha)$, one arrives at $\Pi_i^\mu \left. \frac{\delta I[A_1]}{\delta A_{1i}(\xi)} \right|_{A_1 = A_1(x, \theta, \varphi)} = 0$, and this leads to $\left. \frac{\delta I[A_1]}{\delta A_{1i}(\xi)} \right|_{A_1 = A_1(x, \theta, \varphi)} = 0$

---

[8]Clearly, eq. (18) shows that the standard superspace three-form $\mathcal{A}_3$ cannot be invariant under the transformations of standard supersymmetry. The $\mathcal{A}_3$ here is, up to a factor, the ω_3 in eq. (31) and corresponds to the $\mathcal{A}_3$ form in the case of curved susperspace.

provided that the induced metric on the worldvolume is nondegenerate, as is always the case in tensionful brane theory. But this last equation is one of the equations for $I[x, \theta, A_1]$. If we now substitute it in the equations for $I[x, \theta, A_1(x, \theta, \varphi)]$, and use the fact that the variations through $A_1(x, \theta, \varphi)$ are proportional to $\left.\frac{\delta I[A_1]}{\delta A_i(\xi)}\right|_{A_1 = A_1(x, \theta, \varphi)}$ due to the chain rule, we recover the remaining equations of $I[x, \theta, A_1]$.

Moreover, since in $I[x, \theta, A_1(x, \theta, \varphi)]$ all the new variables appear inside A_1, the extra degrees of freedom corresponding to them have to be reduced by a gauge symmetry to those of the customary BI field $A_{1i}(\xi)$. Since $\frac{\delta I}{\delta \varphi_\alpha} = 0$ is itself a Noether identity (which follows by looking at the E-L equations as above), by the second Noether theorem there is a gauge symmetry allowing us to remove φ_α entirely. We also note that the D equations $\left.\frac{\delta I[A_1]}{\delta \varphi_\mu}\right|_{A_1 = A_1(x, \theta, \varphi)}$ produce only $(p + 1)$ independent ones $\frac{\delta I[A]}{\delta A_{1i}(\xi)} = 0$ and, consequently, the remaining $[D - (p + 1)]$ equations are Noether identities. Hence, of the D extra bosonic degrees of freedom variables introduced by φ_μ in $A_{1i}(x, \theta, \varphi)$, $[D - (p+1)]$ may be eliminated by a gauge transformation. To see this explicitly, let us set $\varphi_\alpha = 0$ and write $A_{1i}(x, \varphi) = \varphi_\mu \partial_i x^\mu$ in a local gauge such that $x^0 = \tau, x^1 = \sigma^1, \ldots, x^p = \sigma^p$ and let $x^K(\xi)$ be the remaining x's. Then $A_i(\varphi, x, \theta) = \varphi_i(\xi) + \varphi_K(\xi) \partial_i x^K(\xi)$, $K = (p + 1), \ldots, D - 1$. Thus, $A_{1i}(\varphi, x, \theta)$, and therefore the action, remains invariant under the following set of $D - (p + 1)$ gauge transformations:

$$\delta \varphi_K(\xi) = \alpha_K(\xi) \quad , \quad \delta \varphi_i(\xi) = -\alpha_K(\xi) \partial_i x^K(\xi) \quad . \tag{21}$$

By taking $\alpha_K = \varphi_K$ we find that $\varphi_K = 0$ so that $A_i(\xi) = \varphi_i(\xi)$. Thus, the actual number of degrees of freedom is $(p + 1)$, and no new dynamical ones are added by assuming the compositeness $A_{1i}(x, \theta, \varphi)$ of the BI fields. The case of the M5 brane can be treated similarly [22].

We show next that an analogous mechanism is at work in D=11 supergravity when it is expressed in terms of a composite three form A_3 by using the coordinates of a suitably enlarged superspace.

4 $D = 11$ supergravity and composite nature of the A_3 field

We turn now to $D = 11$ supergravity, a different theory that nevertheless presents several analogies with the previous discussion on branes. Our aim here is to find a composite structure of the A_3 field of CJS supergravity. This problem is equivalent to that of trivializing a four-cocycle (ω_4 below) for the standard supersymmetry algebra cohomology on a larger superalgebra, so that $\omega_4 = d\tilde{\omega}_3$ where $\tilde{\omega}_3$ is expressed in terms of MC one-forms on the corresponding larger superspace group manifold. In this way A_3 will not be 'external' to the superspace coordinates of the theory and, at the same time, the enlarged superymmetry algebra will reveal the hidden underlying gauge symmetry of CJS supergravity.

The field content of Cremmer-Julia-Scherk $D = 11$ supergravity multiplet is the (unique) $D = 11$ supergravity one

$$(\, e^a(x) \, , \, \psi^\alpha(x) \, , \, A_3(x) \,) \tag{22}$$

where $e^a(x)$ is the *elfbein*, $\psi^\alpha(x)$ (a Majorana spinor) is the gravitino field and $A_3(x)$ is an antisymmetric three-index Abelian gauge field. The first order formulation of $D = 11$ supergravity further requires an initially independent spin connection $\omega^{ab}(x)$.

As is well known, a justification for this set of fields is provided by the on-shell counting of bosonic and fermionic degrees of freedom. By considering the transverse traceless spatial $(D = 11)$ components of g_{ij} and those of A_{ijk}, one finds that e^a has $\frac{(D-2)(D-1)}{2} - 1 = \frac{D(D-3)}{2} = 44$ bosonic d.o.f. and that A_3 has $\binom{D-2}{3} = 84$ bosonic ones; as for ψ^α, it has $\frac{1}{2}2^{[D/2]}(D-3) = \frac{1}{2}32(11-3) = 128$ fermionic d.o.f. Thus, as it should be the case because of the supersymmetry of the theory, the numbers of bosonic and fermionic d.o.f. match:

$$\sharp \text{ Bosonic d.o.f. } = 44 + 84 = 128 = \ \sharp \text{ Fermionic d.o.f.} \quad .$$

The supergravity one-forms e^a, ψ^α and ω^{ab} generate a free differential algebra[9] (FDA) . This is defined by the expressions for the FDA curvatures

$$\mathbf{R}^a \quad := \quad de^a - e^b \wedge \omega_b{}^a + i\psi^\alpha \wedge \psi^\beta \Gamma^a_{\alpha\beta} \, , \tag{23}$$

$$\mathbf{R}^\alpha \quad := \quad d\psi^\alpha - \frac{1}{4}\psi^\beta \wedge \omega^{ab}\Gamma_{ab\,\beta}{}^\alpha \, , \tag{24}$$

$$\mathbf{R}^{ab} \quad := \quad d\omega^{ab} - \omega^{ac} \wedge \omega_c{}^b \, , \tag{25}$$

where $T^a := De^a = de^a - e^b \wedge \omega_b{}^a$ is the torsion and $\mathbf{R}^{ab}$ coincides with the Riemann curvature, and by the Bianchi identities (which are the consistency/integrability conditions for the FDA).

For vanishing curvatures, $\mathbf{R}^a = 0$, $\mathbf{R}^\alpha = 0$, $\mathbf{R}^{ab} = 0$, eqs. (23) and (24), (25) reduce to the MC equations for the superPoincaré algebra. Removing the unessential Lorentz part one arrives to the MC equations for the graded translations (supersymmetry) algebra $\mathfrak{E}^{(11|32)}$ ($\mathfrak{E}^{(D|n)}$ in general) (see footnote 5 for conventions)

$$de^a = -i\psi^\alpha \wedge \psi^\beta \Gamma^a_{\alpha\beta} \, , \qquad d\psi^\alpha = 0 \, , \tag{26}$$

which correspond to the supersymmetry algebra commutation relations,

$$\{Q_\alpha, Q_\beta\} = \Gamma^a_{\alpha\beta}P_a \, , \qquad [P_a, Q_\alpha] = 0 \, , \quad [P_a, P_b] = 0 \, . \tag{27}$$

Eq. (26) is solved by

$$e^a = \Pi^a := dx^a - id\theta^\alpha \Gamma^a_{\alpha\beta}\theta^\beta \, , \qquad \psi^\alpha = \Pi^\alpha := d\theta^\alpha \, . \tag{28}$$

When Π^a, Π^α are considered as forms on the rigid superspace $\Sigma^{(11|32)}$ ($\Sigma^{(D|n)}$ in general) parametrized by $Z^M = (x^a, \theta^\alpha)$, they define the invariant MC forms of the supertranslation

---

[9]In essence, a FDA (introduced in this context in [21] as a *Cartan integrable system*) is an exterior algebra of forms, with constant coefficients, that is closed under the exterior derivative d; see [56, 21, 57, 58].

algebra (27) on the *standard supersymmetry group manifold* $\Sigma^{(11|32)}$ that may be identified with rigid superspace. When e^a and ψ^α are forms on spacetime, the x^a are still spacetime coordinates while the θ^α are Grassmann functions, $\theta^\alpha(x)$, the Volkov-Akulov Goldstone fermions [59]. For one-forms defined on the standard curved superspace, $e^a = dZ^M E_M^a(Z)$, $\psi^\alpha = dZ^M E_M^\alpha(Z)$, $\omega^{ab}(Z) = dZ^M \omega_M^{ab}(Z)$ the FDA (23), (24), (25) with nonvanishing $\mathbf{R}^\alpha$ and $\mathbf{R}^{ab} = R^{ab}$ but vanishing $\mathbf{R}^a = 0$ gives a set of superspace supergravity constraints (which are kinematical or *off-shell* for $N = 1, D = 4$, and *on-shell, i.e.* containing equations of motion among their consequences, for higher D including $D = 11$). Nevertheless, the FDA makes also sense for forms on spacetime, where $e^a = dx^\mu e_\mu^a(x)$ and $\psi^\alpha = dx^\mu \psi_\mu^\alpha(x)$ are the gauge fields for the supertranslations group.

However, the $D = 11$ supermultiplet (22) also includes the three-form A_3, and the previous FDA generated by the one-forms e^a, ψ^α and ω^{ab} has to be completed by the definition of the four–form field strength [21]

$$\mathbf{R}_4 \;\; = \;\; dA_3 + \frac{1}{4}\psi^\alpha \wedge \psi^\beta \wedge e^a \wedge e^b \Gamma_{ab\alpha\beta} \; . \tag{29}$$

Note that, considering the FDA (23), (24), (25), (29) on the $D = 11$ superspace and setting $\mathbf{R}^a = 0$ and $\mathbf{R}_4 = F_4 := 1/4! e^{a_4} \wedge \ldots \wedge e^{a_1} F_{a_1 \ldots a_4}$ one arrives at the original on-shell $D = 11$ superspace supergravity constraints [60, 61] (see also [62, 63]).

Thus, in contrast with the $D = 4$ case, the $D = 11$ supergravity FDA for vanishing curvatures cannot be associated with the MC *one*-forms and equations of a *Lie* superalgebra due to the presence of the *three*-form A_3. On $\Sigma^{(11|32)}$, where one also sets $\mathbf{R}_4 = 0$ by consistency, dA_3 becomes the bosonic four-form

$$a_4 = -\frac{1}{4}\psi^\alpha \wedge \psi^\beta \wedge e^a \wedge e^b \Gamma_{ab\alpha\beta} \; , \tag{30}$$

which corresponds to CE Lie algebra cohomology *four-cocycle* on the standard supersymmetry algebra $\mathfrak{E}^{(11|32)}$, *i.e.*

$$\omega_4 = -\frac{1}{4}\Pi^\alpha \wedge \Pi^\beta \wedge \Pi^a \wedge \Pi^b \Gamma_{ab\alpha\beta} = d\omega_3(x,\theta) \equiv -\frac{1}{4}d\theta^\alpha \wedge d\theta^\beta \wedge \Pi^a \wedge \Pi^b \Gamma_{ab\alpha\beta} \; . \tag{31}$$

This is so because ω_4 is 1) $\Sigma^{(11|32)}$–invariant and 2) closed. The four-cocycle ω_4 is, furthermore, CE non-trivial, since ω_3 cannot be expressed in terms of the $\mathfrak{E}^{(11|32)}$ MC forms: ω_3 is not $\Sigma^{(11|32)}$-invariant. However, it will be seen that there exists [45] a one-parametric family of *extended* superalgebras $\tilde{\mathfrak{E}}(s)$, with MC forms defined on the associated extended superspace group $\tilde{\Sigma}(s)$ manifolds, on which the CE four-cocycle ω_4 becomes trivial *i.e*, $\omega_4 = d\tilde\omega$ and $\tilde\omega$ is made out of $\tilde{\Sigma}(s)$-invariant MC forms. Of course, we have already mentioned another solution for the same mathematical problem in the context of branes, eq. (12), but here we shall concentrate on $\tilde{\mathfrak{E}}(s)$ due to their closer relation with $osp(1|32)$ and with the M-theory superalgebra (itself an expansion, osp(1—32)(2,1,2), of $osp(1|32)$ [40]). Substituting in the expression of $\tilde\omega_3$ the gauge fields for the MC forms, the resulting expression will provide the composite structure of the A_3 form in terms of $\tilde{\mathfrak{E}}(s)$ gauge fields.

Thus, formulated in this way, the problem of writing the A_3 field in terms of one-form fields is, like the construction of WZ invariant terms for branes or the search for an (enlarged) superspace origin of the BI fields, purely geometrical: it reduces to a problem of Lie superalgebra cohomology. It is equivalent, in the spirit of the *enlarged superspace coordinates/fields correspondence*, to looking for an enlarged supergroup manifold $\tilde\Sigma$ on which one can find a suitable invariant three-form $\tilde\omega_3$ (corresponding to A_3) written in terms of products of $\tilde{\mathfrak{E}}$ MC forms on $\tilde\Sigma$ (which will give rise to the one-form gauge fields). The three-form $\tilde\omega_3$ will necessarily depend on the coordinates of the generalized (extended) superspace group manifold $\tilde\Sigma$; in contrast, the original $\omega_3 = \omega_3(x^a, \theta^\alpha)$ depends on the coordinates of standard superspace $\Sigma^{(11|32)}$. The MC equations of the enlarged superspace algebra $\tilde{\mathfrak{E}}$ can be 'softened' by adding the appropriate curvatures. The resulting gauge FDA for the 'soft' forms over $D = 11$ spacetime will then describe a $D = 11$ supergravity theory in which A_3 is a *composite*, not elementary, field.

To finish this section, we remark that, although the composite nature of A_3 and the superspace origin of the worldvolume fields in the D-branes and M5 brane depend on the same mathematical problem as stated above, the relevant objects involved in those two problems are different: whereas $\tilde\omega_3$ above, being constructed entirely in terms of the MC forms of certain enlarged supergroups is invariant under its transformations, the *e.g.* BI fields expressed through the coordinates of the enlarged superspaces are not invariant under the corresponding enlarged supersymmetry (see eqs. (19) and (20)); only the forms $\mathcal{F}$ and H are invariant.

5 Trivialization of the CE four-cocycle

Let us describe now the solution of the trivialization problem just described. We shall first write down the algebras suitable to this end, and then the expression for $\tilde\omega_3$, which also gives the composite structure of A_3. At the end of this section, we shall specialize the results for a particularly simple case.

5.1 A family of extended superalgebras $\tilde{\mathfrak{E}}(s)$

The three-form A_3 of the $D = 11$ supergravity FDA may be written in terms of one-forms by introducing *two* new *bosonic* tensorial one-forms, $B^{a_1 a_2}$, $B^{a_1 \dots a_5}$, and *one* new *fermionic* spinorial one-form, η^α, that obey the FDA equations (23)–(25), (29) plus

$$\mathcal{B}_2^{a_1 a_2} = DB^{a_1 a_2} + \psi^\alpha \wedge \psi^\beta \, \Gamma_{\alpha\beta}^{a_1 a_2} \,, \tag{32}$$

$$\mathcal{B}_2^{a_1 \dots a_5} = DB^{a_1 \dots a_5} + i\psi^\alpha \wedge \psi^\beta \, \Gamma_{\alpha\beta}^{a_1 \dots a_5} \,, \tag{33}$$

$$\begin{aligned}
\mathcal{B}_2^\alpha = {} & D\eta^\alpha - i\, \delta\, e^a \wedge \psi^\beta \Gamma_{a\,\beta}{}^\alpha \\
& - \gamma_1\, B^{ab} \wedge \psi^\beta \Gamma_{ab\,\beta}{}^\alpha - i\, \gamma_2\, B^{a_1 \dots a_5} \wedge \psi^\beta \Gamma_{a_1 \dots a_5 \beta}{}^\alpha
\end{aligned} \tag{34}$$

where γ_1, γ_2 and δ are parameters (that are related by eq. (40) below).

For vanishing curvatures (and ignoring the spin connection) the above FDA reduces to the MC equations

$$de^a = -i\psi^\alpha \wedge \psi^\beta \Gamma^a_{\alpha\beta}, \quad d\psi^\alpha = 0,$$ (35)

$$dB^{a_1a_2} = -\psi^\alpha \wedge \psi^\beta \Gamma^{a_1a_2}_{\alpha\beta}, \quad dB^{a_1...a_5} = -i\psi^\alpha \wedge \psi^\beta \Gamma^{a_1...a_5}_{\alpha\beta},$$ (36)

$$d\eta^\alpha = \psi^\beta \wedge \left(-i\,\delta\,e^a\Gamma_a - \gamma_1\,B^{ab}\Gamma_{ab} - i\,\gamma_2\,B^{a_1...a_5}\Gamma_{a_1...a_5}\right)_\beta^{\ \alpha},$$ (37)

which correspond to the $D = 11$ superalgebra commutators

$$\{Q_\alpha, Q_\beta\} = P_{\alpha\beta} := \Gamma^a_{\alpha\beta}P_a + i\Gamma^{a_1a_2}_{\alpha\beta}Z_{a_1a_2} + \Gamma^{a_1...a_5}_{\alpha\beta}Z_{a_1...a_5},$$ (38)

$$[P_a, Q_\alpha] = \delta\,\Gamma_a{}_\alpha{}^\beta Q'_\beta,$$

$$[Z_{a_1a_2}, Q_\alpha] = i\gamma_1\Gamma_{a_1a_2}{}_\alpha{}^\beta Q'_\beta, \quad [Z_{a_1...a_5}, Q_\alpha] = \gamma_2\Gamma_{a_1...a_5}{}_\alpha{}^\beta Q'_\beta.$$ (39)

The constants δ, γ_1, γ_2 are clearly restricted by the Jacobi identities, which require

$$\delta + 10\gamma_1 - 6!\gamma_2 = 0.$$ (40)

One non-vanishing parameter (*e.g.*, γ_1) can be removed by rescaling the new fermionic generator Q'_α and it is thus inessential. As a result, eqs. (38)–(40) describe, effectively, a *one-parameter family* of Lie superalgebras, denoted $\tilde{\mathfrak{E}}(s)$. The parameter s may be introduced, *e.g.* through

$$s := \frac{\delta}{2\gamma_1} - 1 \quad \Rightarrow \quad \begin{cases} \delta = 2\gamma_1(s+1), \\ \gamma_2 = 2\gamma_1(\frac{s}{6!} + \frac{1}{5!}). \end{cases}$$ (41)

In this parametrization the element corresponding to the case $\gamma_1 = 0$ may be included as the $\gamma_1 \to 0$ limit with $\gamma_1 s \to \delta/2 \neq 0$. This implies that the corresponding algebra (labelled $\tilde{\mathfrak{E}}(\infty)$) is a regular member of the family.

In terms of s, eqs. (39) read:

$$[P_a, Q_\alpha] = 2\gamma_1(s+1)\,\Gamma_a{}_\alpha{}^\beta Q'_\beta,$$

$$[Z_{a_1a_2}, Q_\alpha] = i\gamma_1\Gamma_{a_1a_2}{}_\alpha{}^\beta Q'_\beta,$$

$$[Z_{a_1...a_5}, Q_\alpha] = 2\gamma_1(\tfrac{s}{6!} + \tfrac{1}{5!})\Gamma_{a_1...a_5}{}_\alpha{}^\beta Q'_\beta.$$ (42)

The family $\tilde{\mathfrak{E}}(s)$ is equivalently defined by its MC equations

$$de^a = -i\psi^\alpha \wedge \psi^\beta \Gamma^a_{\alpha\beta}, \quad d\psi^\alpha = 0,$$

$$dB^{a_1a_2} = -\psi^\alpha \wedge \psi^\beta \Gamma^{a_1a_2}_{\alpha\beta},$$

$$dB^{a_1...a_5} = -i\psi^\alpha \wedge \psi^\beta \Gamma^{a_1...a_5}_{\alpha\beta},$$

$$d\eta^\alpha = -2\gamma_1\psi^\beta \wedge \left(i\,(s+1)\,e^a\Gamma_a{}_\beta{}^\alpha\right.$$

$$\left. + \frac{1}{2}B^{ab}\Gamma_{ab}{}_\beta{}^\alpha + i\left(\frac{s}{6!} + \frac{1}{5!}\right)B^{a_1...a_5}\Gamma_{a_1...a_5}{}_\beta{}^\alpha\right).$$ (43)

14

The $s = 0$ is a special case. The $\tilde{\mathfrak{E}}(0)$ superalgebra is given by

$$\{Q_\alpha, Q_\beta\} = P_{\alpha\beta} , \quad [P_{\alpha\beta}, Q_\gamma] = 64 \, \gamma_1 \, C_{\gamma(\alpha} Q'_{\beta)} , \tag{44}$$

which are obtained from eqs. (42) with $s = 0$ by using the Fierz identity

$$\delta_{(\alpha}{}^\gamma \delta_{\beta)}{}^\delta = \frac{1}{32} \left(\Gamma^a_{\alpha\beta} \Gamma^{\gamma\delta}_a - \frac{1}{2} \Gamma^{a_1 a_2}{}_{\alpha\beta} \Gamma_{a_1 a_2}{}^{\gamma\delta} + \frac{1}{5!} \Gamma^{a_1 \ldots a_5}{}_{\alpha\beta} \Gamma_{a_1 \ldots a_5}{}^{\gamma\delta} \right) .$$

Equivalently, collecting the bosonic one-forms e^a, $B^{a_1 a_2}$, $B^{a_1 \cdots a_5}$ in (43) for $s = 0$ in a symmetric spin-tensor one-form $\mathcal{E}^{\alpha\beta}$,

$$\mathcal{E}^{\alpha\beta} = \frac{1}{32} \left(e^a \Gamma^{\alpha\beta}_a - \frac{i}{2} B^{a_1 a_2} \Gamma_{a_1 a_2}{}^{\alpha\beta} + \frac{1}{5!} B^{a_1 \ldots a_5} \Gamma_{a_1 \ldots a_5}{}^{\alpha\beta} \right) , \tag{45}$$

the MC equations of $\tilde{\mathfrak{E}}(0)$ can be written as

$$d\mathcal{E}^{\alpha\beta} = -i\psi^\alpha \wedge \psi^\beta , \quad d\psi^\alpha = 0 , \quad d\eta^\alpha = -64 i \gamma_1 \, \psi^\beta \wedge \mathcal{E}_\beta{}^\alpha ; \tag{46}$$

in the form given by eqs. (44) or (46) the $Sp(32)$ automorphism symmetry of $\tilde{\mathfrak{E}}(0)$ becomes manifest.

For our purposes, the relevant features of the $\tilde{\mathfrak{E}}(s)$ superalgebras are the following:

1. For $s \neq 0$, the $\tilde{\mathfrak{E}}(s)$ may be considered as deformations of $\tilde{\mathfrak{E}}(0)$.

2. The automorphism group of $\tilde{\mathfrak{E}}(0)$ is $Sp(32)$ while, for $s \neq 0$, $\tilde{\mathfrak{E}}(s)$ has the smaller $SO(1, 10)$ group of automorphisms. Hence, the groups that generalize the super-Poincaré group $\Sigma^{(11|32)} \rtimes SO(1, 10)$, are given by the following semidirect products

   - $\tilde{\Sigma}(s) \rtimes SO(1, 10)$, $s \neq 0$, and
   - $\tilde{\Sigma}(0) \rtimes SO(1, 10) \approx Osp(1|32)(2, 3, 2)$,
   - $\tilde{\Sigma}(0) \rtimes Sp(32) \approx Osp(1|32)(2, 3)$,

where the last two right hand sides denote the appropriate *expansions* of $OSp(1|32)$ (see later).

5.2 Trivialization of ω_4

To trivialize the CE four-cocycle $\omega_4 = -\frac{1}{4} \Pi^\alpha \wedge \Pi^\beta \wedge \Pi^a \wedge \Pi^b \Gamma_{ab\alpha\beta} = d\omega_3$ (eq.(31)), over the $\tilde{\mathfrak{E}}(s)$ enlarged superalgebra one considers first the most general ansatz that expresses the three–form A_3 in terms of combinations of wedge products of the one-forms e^a, ψ^α;

$B^{a_1 a_2}$, $B^{a_1 \dots a_5}$, η^α, which are assumed to satisfy the MC equations (35)–(37). Using the same notation for MC forms and fields here and below, we write

$$
\begin{aligned}
4A_3 \;=\; & \lambda B^{ab} \wedge e_a \wedge e_b \;-\; \alpha_1 B_{ab} \wedge B^b{}_c \wedge B^{ca} \\
& -\; \alpha_2 B_{b_1 a_1 \dots a_4} \wedge B^{b_1}{}_{b_2} \wedge B^{b_2 a_1 \dots a_4} \\
& -\; \alpha_3 \epsilon_{a_1 \dots a_5 b_1 \dots b_5 c} B^{a_1 \dots a_5} \wedge B^{b_1 \dots b_5} \wedge e^c \\
& -\; \alpha_4 \epsilon_{a_1 \dots a_6 b_1 \dots b_5} B^{a_1 a_2 a_3}{}_{c_1 c_2} \wedge B^{a_4 a_5 a_6 c_1 c_2} \wedge B^{b_1 \dots b_5} \\
& -\; 2i\psi^\beta \wedge \eta^\alpha \wedge \big(\beta_1\, e^a \Gamma_{a\alpha\beta} \\
& \qquad\qquad\qquad -i\beta_2\, B^{ab}\Gamma_{ab\,\alpha\beta} + \beta_3\, B^{abcde}\Gamma_{abcde\,\alpha\beta} \big) \ .
\end{aligned}
\tag{47}
$$

The problem is now to find the values of the constants $\alpha_1, \dots, \alpha_4$, $\beta_1, \dots, \beta_3$ and λ, such that eq. (30), $dA_3 = a_4 = -\frac{1}{4}\psi^\alpha \wedge \psi^\beta \wedge e^a \wedge e^b \Gamma_{ab\alpha\beta}$, is fulfilled. This produces a set of equations for the constants α_1, …, β_3 and λ that includes δ, γ_1 and γ_2 as parameters:

$$
\begin{array}{ll}
\lambda - 2\delta\beta_1 = 1 \,, & \lambda - 2\gamma_1\beta_1 - 2\delta\beta_2 = 0 \,, \\
3\alpha_1 + 8\gamma_1\beta_2 = 0 \,, & \alpha_2 - 10\gamma_1\beta_3 - 10\gamma_2\beta_2 = 0 \,, \\
5!\alpha_3 - \delta\beta_3 - \gamma_2\beta_1 = 0 \,, & \alpha_2 - 5!\,10\gamma_2\beta_3 = 0 \,, \\
\alpha_3 - 2\gamma_2\beta_3 = 0 \,, & 3\alpha_4 + 10\gamma_2\beta_3 = 0 \,.
\end{array}
$$

$$\tag{48}$$

This system has a nontrivial solution for

$$
\Delta = (2\gamma_1 - \delta)^2 = 4s^2\gamma_1^2 \neq 0 \quad \Longleftrightarrow \quad s \neq 0 \,,
\tag{49}
$$

which in terms of the parameter s reads [45]

$$
\lambda = \tfrac{1}{5}\,\frac{s^2 + 2s + 6}{s^2} \,,
$$
$$
\beta_1 = -\frac{1}{10\gamma_1}\frac{2s - 3}{s^2} \,, \quad \beta_2 = \frac{1}{20\gamma_1}\frac{s + 3}{s^2} \,, \quad \beta_3 = \frac{3}{10 \cdot 6!\gamma_1}\frac{s + 6}{s^2} \,,
$$
$$
\alpha_1 = -\frac{1}{15}\frac{2s + 6}{s^2} \,, \quad \alpha_2 = \frac{1}{6!}\frac{(s+6)^2}{s^2} \,, \quad \alpha_3 = \frac{1}{5 \cdot 5!}\alpha_2 \,, \quad \alpha_4 = -\frac{1}{9 \cdot 5!}\alpha_2 \,;
\tag{50}
$$

note that $\alpha_{2,3,4} \propto (s + 6)$ and that all denominators depend on s^2. Thus, ω_4 can be trivialized ($\omega_4 = d\tilde\omega_3$) over $\tilde{\mathfrak{E}}(s)$ when $s \neq 0$; the impossibility of doing it over $\tilde{\mathfrak{E}}(0)$ may be related to the fact that precisely $\tilde{\mathfrak{E}}(0)$ has an enhanced automorphism symmetry, $Sp(32)$. This implies that the A_3 field can be considered as a composite of the one-form gauge fields of any of the $\tilde\Sigma(s)$ with $s \neq 0$, and that A_3 is given by eq. (47) for the values (50) of $\alpha_1, \dots, \beta_3, \lambda$. Thus, the hidden gauge symmetry of $D = 11$ supergravity can be associated with any of the $\tilde\Sigma(s) \rtimes SO(1, 10)$ supergroups.

The two particular solutions of D'Auria-Fré for A_3 are recovered by adjusting δ, γ_1 in eq. (50) so that $\lambda = 1$, which was the starting point of [21]. These correspond to $\tilde{\mathfrak{E}}(3/2)$, given by the parameter values

$$
\delta = 5\gamma_1 \neq 0 \,, \quad \gamma_2 = \tfrac{\gamma_1}{2 \cdot 4!} \,,
$$
$$
\lambda = 1 \,, \quad \beta_1 = 0 \,, \quad \beta_2 = \tfrac{1}{10\gamma_1} \,, \quad \beta_3 = \tfrac{1}{6!\,\gamma_1} \,,
$$
$$
\alpha_1 = -\tfrac{4}{15} \,, \quad \alpha_2 = \tfrac{25}{6!} \,, \quad \alpha_3 = \tfrac{1}{6!\,4!} \,, \quad \alpha_4 = -\tfrac{1}{54\,(4!)^2} \,,
\tag{51}
$$

and to $\tilde{\mathfrak{E}}(-1)$, for which

$$\delta = 0 \ , \ \gamma_1 \neq 0 \ , \ \gamma_2 = \tfrac{\gamma_1}{3 \cdot 4!} \ ,$$
$$\lambda = 1 \ , \ \beta_1 = \tfrac{1}{2\gamma_1} \ , \ \beta_2 \tfrac{1}{10\gamma_1} \ , \ \beta_3 = \tfrac{1}{4 \cdot 5! \, \gamma_1} \ ,$$
$$\alpha_1 = -\tfrac{4}{15} \ , \ \alpha_2 \tfrac{25}{6!} \ , \ \alpha_3 = \tfrac{1}{6! \, 4!} \ , \ \alpha_4 = -\tfrac{1}{54 \, (4!)^2} \ . \tag{52}$$

5.3 The minimal solution $\mathfrak{E}_{min}$

A specially simple trivialization of ω_4 is achieved for the superalgebra $\tilde{\mathfrak{E}}(-6)$, characterized by

$$\tilde{\mathfrak{E}}(-6) \ : \qquad \delta \neq 0 \ , \qquad \delta = -10\gamma_1 \ , \ \gamma_2 = 0 \ . \tag{53}$$

In $\tilde{\mathfrak{E}}(-6)$ the generator $Z_{a_1 \ldots a_5}$ is central (see eq. (42)) and does not play any rôle in the trivialization of the ω_4 cocycle. Furthermore, eqs. (38)–(40) allow us to use instead the $\tilde{\mathfrak{E}}_{min}$ superalgebra whose central extension by the generator $Z_{a_1 \ldots a_5}$ gives $\tilde{\mathfrak{E}}(-6)$. $\mathfrak{E}_{min}$ is the $(66 + 64)$-dimensional superalgebra $\tilde{\mathfrak{E}}^{(66|32+32)}$,

$$\mathfrak{E}_{min} : \qquad \{Q_\alpha, Q_\beta\} = \Gamma^a_{\alpha\beta} P_a + i \Gamma^{a_1 a_2}_{\alpha\beta} Z_{a_1 a_2} \ , \tag{54}$$
$$[P_a, Q_\alpha] = -10\gamma_1 \, \Gamma_{a\,\alpha}{}^\beta Q'_\beta \ , \quad [Z_{a_1 a_2}, Q_\alpha] = i\gamma_1 \Gamma_{a_1 a_2\,\alpha}{}^\beta Q'_\beta \ , \tag{55}$$

associated with the most economic $\tilde{\Sigma}_{min} \equiv \Sigma^{(66|32+32)}$ generalized supertranslation group that trivializes ω_4.

Using the values of eq. (53) in eq. (50) we get

$$\lambda = \tfrac{1}{6} \ , \ \beta_1 = \tfrac{1}{4! \gamma_1} \ , \ \beta_2 = -\tfrac{1}{2 \cdot 5! \gamma_1} \ , \ \beta_3 = 0 \ ,$$
$$\alpha_1 = \tfrac{1}{90} \ , \ \alpha_2 = 0 \ , \ \alpha_3 = 0 \ , \ \alpha_4 = 0 \ . \tag{56}$$

Then, all the $B^{a_1 \ldots a_5}$ terms in A_3, eq. (47), are zero. This simplifies the expression for A_3 drastically,

$$A_3 \ = \ \frac{1}{4!} B^{ab} \wedge e_a \wedge e_b \ - \ \frac{1}{3 \cdot 5!} B_{ab} \wedge B^b{}_c \wedge B^{ca}$$
$$- \ \frac{i}{4 \cdot 5! \, \gamma_1} \psi^\beta \wedge \eta^\alpha \wedge \left(10 \, e^a \Gamma_{a\alpha\beta} + i \, B^{ab} \Gamma_{ab\,\alpha\beta} \right) \ . \tag{57}$$

Thus, $\Sigma^{(66|32+32)}$ can be regarded as a *minimal* underlying gauge supergroup of $D = 11$ supergravity.

6 Degrees of freedom in $D{=}11$ supergravity with a composite A_3 and extra gauge gauge symmetries

It remains to be checked that, as in the case of the BI fields of the D-branes, the composite nature of A_3 does not change the supergravity degrees of freedom. Let us first recall that,

in standard CJS supergravity, $e_\mu^a(x)$ has $\frac{(D-2)(D-1)}{2} - 1 = \frac{D(D-3)}{2} = 11^2 - 55_{Lorentz} - 2 \times 11_{Diff} = 44$ degrees of freedom; $\psi_\mu^\alpha(x)$ has $(9 \times 32 - 32) \times \frac{1}{2} = 128$; and $A_{\mu\nu\rho}(x)$ has $\binom{9}{3} = \binom{11}{3} - \binom{10}{2} - \binom{9}{2} = 84 = 165_{\# \text{ of components}} - 45_{(\#\text{gauge symm.} - \#\text{null vectors})} - 36_{\#\text{residual gauge symm.}}$.

Now, let us consider a composite A_3 in the CJS supergravity action [29] *i.e.*, by substituting

$$A_3 = A_3(B_1^{ab}, B_1^{a_1...a_5}, \eta_{1\alpha} \; ; \; e^a, \psi^\alpha) , \tag{58}$$

as given by eqs. (47) and (50), for the original three-form field in that action. Naively assuming standard linearized equations and the usual 'group theoretical' gauge symmetry transformations for the new fields,

$$\delta B_\mu^{ab} = \partial_\mu \alpha^{ab} + \dots \; ; \quad \delta B_\mu^{a_1...a_5} = \partial_\mu \alpha^{a_1...a_5} + \dots \; ; \quad \delta\eta_{\mu\alpha} = \partial_\mu \varepsilon_\alpha' + \dots ,$$

the sum of the components of B_μ^{ab} $[9 \times \binom{11}{2} = 495]$, the components of $B_\mu^{a_1...a_5}$ $[9 \times \binom{11}{5} = 4158]$ and those of $\eta_{\mu\alpha}$ [128, as for ψ_μ^α] would give a huge number of 'new' degrees of freedom for the gauge invariant theory with the additional fields. Moreover, the bosonic and fermionic degrees of freedom would not match.

However, the 'new' fields B_1^{ab}, $B_1^{a_1...a_5}$, $\eta_{1\alpha}$ enter in the CJS supergravity action only through the composite $A_3(...)$ three form field and, as a result, the theory possesses *extra gauge symmetries*. Clearly, these are the transformations of the 'new' fields that leave A_3 invariant,

$$\delta B_\mu^{ab} = \partial_\mu \alpha^{ab} + \beta^{\left(\begin{smallmatrix}\square\square\\\square\end{smallmatrix}\right)}{}_\mu^{ab} + \dots$$
$$\delta B_\mu^{a_1...a_5} = \partial_\mu \alpha^{a_1...a_5} + \beta_\mu^{a_1...a_5} + \dots$$
$$\delta\eta_{\mu\alpha} = \partial_\mu \varepsilon_\alpha' + \beta_{\mu\alpha} + \dots .$$

They reduce to 84 the number of B_μ^{ab} degrees of freedom and to zero those of the remaining new fields since, diagrammatically (note that $\#\begin{smallmatrix}\square\square\\\square\end{smallmatrix} = D(D^2 - 1)/3 = 440$)

$$B_{c\,ab} \sim \square \otimes \begin{smallmatrix}\square\\\square\end{smallmatrix} = 11 \times 55 = 605 = \begin{smallmatrix}\square\square\\\square\end{smallmatrix} \oplus \begin{smallmatrix}\square\\\square\\\square\end{smallmatrix} = 440 + 165 , \tag{59}$$

and the equations of motion (which are the standard ones but with a composite A_3), when linearized, affect only the antisymmetric part $B_{[\mu\nu\rho]}$ of B_ν^{ab}. In this way, the antisymmetric 165-dimensional part simulates the fundamental A_3; the mixed symmetry 440-dimensional part of B_ν^{ab} as well as $B_\mu^{a_1...a_5}$ and $\eta_{\mu\alpha}$ are pure gauge and do not have independent equations of motion in the CJS action with a composite A_3. Thus,

$$\# \; d.o.f. \text{ with fundamental } A_3 = \# \; d.o.f. \text{ with composite } A_3,$$

as stated.

7 The special element $\tilde{\mathfrak{E}}(0)$ as an algebra expansion

We have seen that the superalgebra $\tilde{\mathfrak{E}}(0)$, although it does not trivialize ω_4, may be considered as a 'parent' superalgebra for the hidden symmetries of $D = 11$ supergravity in the sense that it gives rise to the family $\tilde{\mathfrak{E}}(s)$ of superalgebras that do trivialize the standard supersymmetry algebra $\mathfrak{E}$ four-cocycle. All the corresponding $\tilde{\Sigma}(s)$ enlarged superspace groups, $s \neq 0$, may be considered as deformations of $\tilde{\Sigma}(0)$. We shall now characterize the parent algebra $\tilde{\mathfrak{E}}(0)$ in terms of Lie algebra expansions. With this aim, we first review briefly the expansion method [40, 39] for the case which is of special interest here.

7.1 The algebra expansion method

Let G be a Lie group, of local coordinates g^i, $\mathcal{G}$ its Lie algebra and $\mathcal{G}^*$ its dual coalgebra. Let $\mathcal{G}$ admit, say, the splitting $\mathcal{G} = V_0 \oplus V_1 \oplus V_2$, where V_0, V_2 (V_1), are even (odd) subspaces of dimensions $\dim V_p$, $p = 0, 1, 2$. Further, let V_0 be a subalgebra of $\mathcal{G}$ and $[V_1, V_1] \subset V_0 \oplus V_2$, $[V_2, V_2] \subset V_0 \oplus V_2$ (details for the general theory are given in [40]). Then, the rescaling of the group parameters $g^{i_p} \to \lambda^p g^{i_p}$, $i_p = 1, \ldots, \dim V_p$, allows us to expand the one-forms $\omega^{i_p}(\lambda, g)$, obtained from the algebra MC forms $\omega^{i_p}(g)$ that define a basis of the dual subspaces $V_p{}^*$, as a series in λ,

$$\omega^{i_p}(\lambda) = \lambda^p \omega^{i_p,p} + \lambda^{p+2} \omega^{i_p,p+2} + \lambda^{p+4} \omega^{i_p,p+4} + \ldots = \sum_{\alpha_p} \lambda^{\alpha_p} \omega^{i_p,\alpha_p} \ \ (p = 0, 1, 2) \ . \quad (60)$$

The different powers of lambda are a consequence of the above assumptions on the subspaces V_p, and follow from the fact that the canonical form $\theta(g)$ on a Lie group G is given by $\theta(g) = g^{-1}dg = \omega^i X_i$, where $g = exp\, g^i X_i$ and X_i are the generators of the algebra $\mathcal{G}$ of G. The insertion of these series expansions into the MC equations of the original algebra $\mathcal{G}$,

$$d\omega^{i_p} = -\tfrac{1}{2} c^{i_p}_{j_q k_s}\, \omega^{j_q} \wedge \omega^{k_s}$$
$$(p, q, s = 0, 1, 2 \ ; \ i_{p,q,s} = 1, 2, \ldots, \dim V_{p,q,s}) \ , \quad (61)$$

produces, identifying the terms with the same order in λ, the following set of equations

$$d\omega^{i_p,\alpha_p} = -\tfrac{1}{2} C^{i_p,\alpha_p}_{j_q,\beta_q\, k_s,\gamma_s}\, \omega^{j_q,\beta_q} \wedge \omega^{k_s,\gamma_s} \ ,$$
$$C^{i_p,\alpha_p}_{j_q,\beta_q\, k_s,\gamma_s} = \begin{cases} 0, & \text{if } \beta_q + \gamma_s \neq \alpha_p \\ c^{i_p}_{j_q k_s}, & \text{if } \beta_q + \gamma_s = \alpha_p \end{cases}$$
$$(\alpha_p, \beta_p, \gamma_p = p, p+2, \ldots) \quad . \quad (62)$$

The question now is how to retain consistently a number of ω^{i_p,α_p} so that the equations above correspond to the MC equations of a new, by construction expanded, algebra. Cutting the expansions of the $\omega^{i_p}(\lambda)$ at certain orders $\alpha_p = N_p$, $p = 0, 1, 2$, one finds that

eqs. (62) for $\alpha_p = p, \ldots, N_p$ will provide the MC equations of a new finite-dimensional Lie algebra provided the chosen orders satisfy the conditions

$$N_0 = N_1 + 1 = N_2 \qquad \text{or} \qquad N_0 = N_1 - 1 = N_2 \qquad \text{or} \qquad N_0 = N_1 - 1 = N_2 - 2 \ . \tag{63}$$

These conditions guarantee that for the selected set of ω^{i_p,α_p}'s, eqs. (62) do not include any ω^{i_p,α_p} outside this set and that, accordingly, define new algebras [39, 40] by becoming their MC equations. These algebras, denoted $\mathcal{G}(N_0, N_1, N_2)$ in obvious notation, are called *expansions* of $\mathcal{G}$; in general, their dimension is larger than that of the original algebra $\mathcal{G}$. They also include, as a particular case and for a specific value of (N_0, N_1, N_2), the generalized Wigner-İnönü contractions [40], in which case the dimension does not change.

The dimension of the expanded $\mathcal{G}(N_0, N_1, N_2)$ algebras is given by

$$\begin{aligned} \dim \mathcal{G}(N_0, N_1, N_2) \ &= \ [(N_0 + 2)/2] \dim V_0 + [(N_1 + 1)/2] \dim V_1 \\ &\quad + [N_2/2] \dim V_2 \quad . \end{aligned} \tag{64}$$

7.2 $\ \tilde{\Sigma}(0) \otimes SO(1, 10)$ as the expansion $OSp(1|32)(2, 3, 2)$

Let us now consider the orthosymplectic algebra $osp(1|32)$, of dimension 560, defined by the MC equations

$$\begin{aligned} d\rho^{\alpha\beta} \ &= \ -i\rho^{\alpha\gamma} \wedge \rho_\gamma{}^\beta - i\nu^\alpha \wedge \nu^\beta \ , \\ d\nu^\alpha \ &= \ -i\nu^\beta \wedge \rho_\beta{}^\alpha \ , \quad \alpha, \beta = 1, \ldots, 32 \quad , \end{aligned} \tag{65}$$

where the 528 $\rho^{\alpha\beta}$ are the bosonic and the 32 ν^α the fermionic MC one-forms.

The decomposition of $\rho_{\alpha\beta}$ as

$$\rho_{\alpha\beta} = \frac{1}{32} \left(\rho^a \Gamma_a - \frac{i}{2} \rho^{ab} \Gamma_{ab} + \frac{1}{5!} \rho^{a_1 \ldots a_5} \Gamma_{a_1 \ldots a_5} \right)_{\alpha\beta} \ , \quad a, b = 0, 1, \ldots, 10 \ . \tag{66}$$

allows us to consider the splitting $osp(1|32) = V_0 \oplus V_1 \oplus V_2$, where

$$\begin{array}{lll} V_0^* & \text{is generated by} \quad \rho^{ab} & (55) \quad , \\ V_1^* & \text{by} \quad \nu^\alpha & (32) \quad , \\ V_2^* & \text{by} \quad \rho^a \text{ and } \rho^{a_1 \ldots a_5} & (11 + 462) \quad . \end{array}$$

The various forms then expand as

$$\begin{array}{llll} V_0^* : & \rho^{ab} = \rho^{ab,0} + \lambda^2 \rho^{ab,2} + \cdots ; & V_1^* : & \nu^\alpha = \lambda \nu^{\alpha,1} + \lambda^3 \nu^{\alpha,3} + \cdots ; \\ V_2^* : & \rho^a = \lambda^2 \rho^{a,2} + \cdots , & \rho^{a_1 \ldots a_5} = \lambda^2 \rho^{a_1 \ldots a_5,2} + \cdots \ . \end{array} \tag{67}$$

Inserting the series into the MC equations and choosing $N_0 = 2$, $N_1 = 3$, $N_2 = 2$ the MC equations of the expansion $osp(1|32)(2,3,2)$ are obtained:

$$
\begin{aligned}
d\rho^{ab,0} &= -\frac{1}{16}\rho^{ac,0} \wedge \rho_c{}^{b,0} \\
d\rho^{a,2} &= -\frac{1}{16}\rho^{b,2} \wedge \rho_b{}^{a,0} - i\nu^{\alpha,1} \wedge \nu^{\beta,1}\Gamma^a_{\alpha\beta} \\
d\rho^{ab,2} &= -\frac{1}{16}\left(\rho^{ac,0} \wedge \rho_c{}^{b,2} + \rho^{ac,2} \wedge \rho_c{}^{b,0}\right) - \nu^{\alpha,1} \wedge \nu^{\beta,1}\Gamma^{ab}_{\alpha\beta} \\
d\rho^{a_1...a_5,2} &= \frac{5}{16}\rho^{b[a_1...a_4|,2} \wedge \rho_b{}^{|a_5],0} - i\nu^{\alpha,1} \wedge \nu^{\beta,1}\Gamma^{a_1...a_5}_{\alpha\beta} \\
d\nu^{\alpha,1} &= -\frac{1}{64}\nu^{\beta,1} \wedge \rho^{ab,0}\Gamma_{ab\beta}{}^\alpha \\
d\nu^{\alpha,3} &= -\frac{1}{64}\nu^{\beta,3} \wedge \rho^{ab,0}\Gamma_{ab\beta}{}^\alpha \\
&\quad -\frac{1}{2}\nu^{\beta,1} \wedge \left(i\rho^{a,2}\Gamma_a + \frac{1}{2}\rho^{ab,2}\Gamma_{ab} + \frac{i}{5!}\rho^{a_1...a_5,2}\Gamma_{a_1...a_5}\right)_\beta{}^\alpha.
\end{aligned}
\tag{68}
$$

With the identifications

$$
\begin{aligned}
\rho^{ab,0} &= -16\omega^{ab} \quad, \quad \rho^{a,2} = e^a \quad, \quad \rho^{ab,2} = B^{ab}, \\
\rho^{a_1\cdots a_5,2} &= B^{a_1\cdots a_5} \quad, \quad \nu^{\alpha,1} = \psi^\alpha \quad, \quad \nu^{\alpha,3} = \eta^\alpha/64\gamma_1 \quad,
\end{aligned}
\tag{69}
$$

and omitting the Lorentz generators ω^{ab} to simplify, these equations read

$$
\begin{aligned}
de^a &= -i\psi^\alpha \wedge \psi^\beta \Gamma^a_{\alpha\beta} \,, \\
dB^{a_1 a_2} &= -\psi^\alpha \wedge \psi^\beta \, \Gamma^{ab}_{\alpha\beta} \,, \\
dB^{a_1...a_5} &= -i\psi^\alpha \wedge \psi^\beta \, \Gamma^{a_1...a_5}_{\alpha\beta} \,, \\
d\psi^\alpha &= 0 \,, \\
d\eta^\alpha &= -2\gamma_1 \cdot \psi^\beta \wedge \left(i\,e^a\Gamma_a + \frac{1}{2}B^{ab}\Gamma_{ab} + \frac{i}{5!}B^{a_1...a_5}\Gamma_{a_1...a_5}\right)_\beta{}^\alpha \,;
\end{aligned}
\tag{70}
$$

the inclusion of ω^{ab} produces the MC equations of the $\tilde{\mathfrak{E}}(0) \otimes\!\!\!\!\!\!- \, so(1,10)$ algebra. Also, one may check that

$$
\begin{aligned}
\dim OSp(1|32)(2,3,2) &= 2\cdot 55 + 2\cdot 32 + 1\cdot 473 = 647 = \\
&= 592 + 55 = \dim\left(\tilde{\Sigma}(0) \otimes\!\!\!\!\!\!- \, SO(1,10)\right) \quad.
\end{aligned}
\tag{71}
$$

7.3 $\tilde{\Sigma}(0) \otimes\!\!\!\!\!\!- \, Sp(32)$ as the expansion $OSp(1|32)(2,3)$

Let us now see that the full $\tilde{\mathfrak{E}}(0) \dotplus sp(32)$ is also an expansion of $osp(1|32)$. Let us consider now the splitting $osp(1|32) = V_0 \oplus V_1$ where V_0 is the bosonic subalgebra, generated by $\rho^{\alpha\beta}$, and V_1 the fermionic part, generated by ν^α. Choosing $N_0 = 2$ and $N_1 = 3$ we obtain the

expansion $osp(1|32)(2,3)$ determined by the MC equations of the one-forms $\rho^{\alpha\beta,0}$, $\rho^{\alpha\beta,2}$, $\nu^{\alpha,1}$, $\nu^{\alpha,3}$:

$$
\begin{aligned}
d\rho^{\alpha\beta,0} &= -i\rho^{\alpha\gamma,0} \wedge \rho_\gamma{}^{\beta,0} \\
d\nu^{\alpha,1} &= -i\nu^{\beta,1} \wedge \rho_\beta{}^{\alpha,0} \\
d\rho^{\alpha\beta,2} &= -i\left(\rho^{\alpha\gamma,0} \wedge \rho_\gamma{}^{\beta,2} + \rho^{\alpha\gamma,2} \wedge \rho_\gamma{}^{\beta,0}\right) - i\nu^{\alpha,1} \wedge \nu^{\beta,1} \\
d\nu^{\alpha,3} &= -i\nu^{\beta,3} \wedge \rho_\beta{}^{\alpha,0} - i\nu^{\beta,1} \wedge \rho_\beta{}^{\alpha,2} \ .
\end{aligned}
\tag{72}
$$

Identifying $\rho^{\alpha\beta,0}$ in eqs. (72) with the $sp(32)$ connection $\Omega^{\alpha\beta}$, eqs, (72) coincide with those of $\tilde{\mathfrak{E}}(0)\dotplus sp(32)$ [eqs. (46)], with the identifications $\rho^{\alpha\beta,2} = \mathcal{E}^{\alpha\beta}$, $\nu^{\alpha,1} = \psi^\alpha$ and $\nu^{\alpha,3} = \eta^\alpha/64\gamma_1$. One can also make a dimensions check:

$$
\begin{aligned}
\dim(\tilde{\mathfrak{E}}(0)\dotplus sp(32)) &= 592\,(528+64) + 528 = 1120 = \\
&= 2\cdot 528 + 2\cdot 32 = \dim(\,osp(1|32)(2,3))
\end{aligned}
\tag{73}
$$

when $N_0 = 2$, $N_1 = 3$ in eq. (64).

8 Conclusions

We have given some reasons in favour of a geometrical *enlarged superspace coordinates/fields correspondence*, both for branes, in which case the correspondence is between the extended superspace coordinates and *worldvolume* fields, and for $D = 11$ CJS supergravity, where the fields are *spacetime* fields.

In the case of *branes*, the new enlarged superspace algebras appear as the result of wishing to have manifestly invariant WZ terms or an (enlarged) superspace origin for all the fields of the theory, including the otherwise 'intrinsically' worldvolume fields of the D-branes (Born-Infeld fields) and of the M5 brane. The CE cohomology arguments that lead to the WZ terms for the scalar p-branes also allow us to caracterize the D-branes as well as the WZ term of the M5-brane. Their actions (apart from the auxiliary field in the M5-brane case) do not contain fields directly defined on the worldvolume[10]; all worldvolume fields are associated to variables of certain enlarged superspaces $\tilde{\Sigma}$. Further, the number of degrees of freedom and the dynamical contents of the E-L equations remain the same once the substitution is made [22].

The fields/coordinates correspondence for $D = 11$ *CJS supergravity* has also to do with trivializing non-trivial CE cocycles. Trivializing the supersymmetry algebra $\mathfrak{E}^{(11|32)}$) CE four–cocycle ω_4 amounts to finding a composite structure for the three–form field A_3 of the standard Cremmer–Julia–Scherk supergravity in terms of one–form gauge fields of $\tilde{\Sigma}(s)$,

---

[10]Although in the (either rigid or non-flat) $D = 11$ covariant M5-brane action [53] the Pasti-Sorokin-Tonin (PST) scalar $a(\xi)$ field [64] is a *worldvolume* field, it is an *auxiliary* one. Further, it was shown in [65] that, when the M5-brane interacts with dynamical supergravity in a duality symmetric formulation, the rôle of the M5-brane auxiliary PST scalar $a(\xi)$ is played by the pull-back $a(x(\xi))$ to $\mathcal{W}$ of the *spacetime* supergravity PST scalar $a(x)$ and is a kind of background field.

$A_3 = A_3(e^a, \psi^\alpha; B^{a_1 a_2}, B^{a_1 \dots a_5}, \eta^\alpha)$. The trivialization of the CE four-cocycle ω_4 may be achieved, for $s \neq 0$, on the one-parametric family of superalgebras $\tilde{\mathfrak{E}}(s)$. These are central extensions of the M-algebra (generated, ignoring the Lorentz part, by P_a, Q_α, Z_{ab}, $Z_{a_1 \dots a_5}$) by an additional fermionic central generator Q'_α. Then, $\omega_4 = d\tilde{\omega}_3(\tilde{Z})$, $\tilde{Z} \in \tilde{\Sigma}$. The Maurer-Cartan forms of $\tilde{\mathfrak{E}}(s)$ can be replaced by *soft* one-forms obeying a free differential algebra with curvatures, and thus *one may treat the standard CJS D=11 supergravity as a gauge FDA of the $\tilde{\Sigma}(s)$ supergroup for any $s \neq 0$*. This fact was known before for the two superalgebras [21] that here correspond to $\tilde{\mathfrak{E}}(3/2)$ and $\tilde{\mathfrak{E}}(-1)$. The novelty of the present results is that, for $s \neq 0$, any of the $\tilde{\Sigma}(s) \oslash SO(1,10)$ supergroups may be equally treated as an underlying gauge supergroup of the $D = 11$ supergravity.

There is a special element in the $\tilde{\mathfrak{E}}(s \neq 0)$ family of trivializations, $\tilde{\mathfrak{E}}(-6)$, for which the $Z_{a_1 \dots a_5}$ generator is central. In this case, the expression for A_3 is particularly simple: it does not involve the one-form $B^{a_1 \dots a_5}$, and $\tilde{\mathfrak{E}}(-6)$ may be reduced to $\mathfrak{E}_{min}$. Thus, the smaller $\tilde{\Sigma}_{min} = \tilde{\Sigma}^{(66|32+32)}$ associated with $\mathfrak{E}_{min}$ may be considered as the minimal underlying gauge supergroup of $D = 11$ CJS supergravity. All other representatives of the family $\tilde{\mathfrak{E}}(s)$ are equivalent, although they are not isomorphic. Their significance might be related to the fact that the field $B^{a_1 \dots a_5}$ is also needed for a coupling to BPS preons [12, 11], the hypothetical basic constituents of M-theory. The presence of a full family of superalgebras $\tilde{\mathfrak{E}}(s \neq 0)$ –rather than a unique one– trivializing the standard $\mathfrak{E}^{(11|32)}$ algebra four–cocycle ω_4, suggests that the obtained underlying gauge symmetries of $D = 11$ supergravity may be incomplete (this is almost certainly the case if one considers the symmetries of M-theory).

The singularity of the $\tilde{\mathfrak{E}}(0)$ case looks a reasonable one; the $\tilde{\Sigma}(0)$ supergroup is special because it possesses an enhanced automorphism symmetry, $Sp(32)$. The full $\tilde{\Sigma}(0) \oslash Sp(32)$, that replaces the $D = 11$ superPoincaré group, is given by the expansion $OSp(1|32)(2,3)$ of $OSp(1|32)$. All other members of the $\tilde{\Sigma}(s \neq 0)$ family have the smaller $SO(1,10)$ automorphism symmetry and are deformations of the $s = 0$ element. Thus, we may conclude that *the underlying gauge group of $D = 11$ supergravity is determined by any element $\tilde{\Sigma}(s \neq 0) \oslash SO(1,10)$, of a one-parametric familiy of nontrivial deformations of* $\tilde{\Sigma}(0) \oslash SO(1,10) \approx OSp(1|32)(2,3,2) \subset \tilde{\Sigma}(0) \oslash Sp(32)$. Furthermore, we see that the number of the extended superspace coordinates are in one-to-one correspondence with the gauge fields entering the theory, and that the additional degrees of freedom may be removed by a gauge transformation. Thus, this may be considered as another example of the conjectured extended superspaces coordinates/fields correspondence principle in which the fields are spacetime fields.

Finally it is known that, unlike its lower dimensional versions, CJS supergravity forbids a cosmological term extension. The reason is cohomological and can be traced to an obstruction produced by the A_3 three-form field [66]. It is natural to ask wether this obstruction remains when A_3 is becomes a composite field.

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